

# Complete disentanglement by partial pure dephasing

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We study the effect of pure dephasing on the entanglement of a pair of two-level subsystems (qubits). We show that partial dephasing induced by a super-Ohmic reservoir, corresponding to well-established properties of confined charge states and phonons in semiconductors, may lead to complete disentanglement. We show also that the disentanglement effect increases with growing distance between the two subsystems.

## I. INTRODUCTION

Entanglement [1, 2] is one of the fundamental constituents of quantum theory. Correlations between the results of appropriately chosen measurements on entangled subsystems cannot be accounted for by any classical (realistic and local) theory [3], precluding the existence of a wide class of hypothetical more fundamental structures underlying the incompleteness of quantum description. Apart from its essential role in our understanding of the quantum world, entanglement is an important resource in quantum information processing [4] where it provides a quantum channel for teleportation [5], superdense coding [6], and distribution of cryptographic keys [7].

In order to manifest genuinely quantum behavior resulting from entanglement a quantum system must maintain phase relations between the components of its quantum superposition state, involving different states of distinct subsystems. Keeping in mind that the subsystems may be separated by a macroscopic distance, one may expect such a non-local superposition state to be extremely fragile to the dephasing effect of the environment. In fact, it has been shown that entanglement of a pair of two-level subsystems tends to decay faster than local coherence [8, 9, 10]. As expected, the decay of entanglement is stronger if the subsystems interact with different environments (which might result from a large spatial separation between them): certain states that show robust entanglement under collective dephasing become disentangled by separate environments [9]. Remarkably, it was shown for different classes of systems [10, 11, 12] that certain states may become separable (completely disentangled) within a final time under conditions that lead to usual, exponential decay of local coherence. Since even partial entanglement of many copies of a bipartite quantum system may still be distilled to a smaller number of maximally entangled systems [13], many quantum information processing and communication tasks may be carried out using partly disentangled systems, as long as there is still some entanglement left. It is therefore essential to understand whether environmental influence leads to the appearance of separability in realistic models of dephasing.

In this paper we study the decay of entanglement

between a pair of spatially localized two-level systems (qubits) under pure dephasing induced by a bosonic bath. In order to capture the physical aspects of disentanglement in real systems, we focus on the specific case of two excitons confined in one or two semiconductor quantum dots and coupled to phonons. In this system, the properties of the coupling and the resulting kinetics of dephasing are well understood. In particular, the experimentally observed evolution of the system may be accounted for by pure dephasing within an independent boson model [14]. In contrast to the properties of the Ohmic model [15], the carrier-phonon dynamics in a real system leads only to *partial* dephasing [16]: non-diagonal elements of the density matrix do not vanish exponentially but rather decay only to a certain finite value (apart from other processes on much longer time scales). This behavior is characteristic of super-Ohmic spectral densities [17, 18] resulting from the actual carrier-phonon couplings and phonon density of states [19]. The effect of pure dephasing on a system of two excitons in a single quantum dot was studied in a recent work [20] in the context of ultrafast optical excitation. Here we describe the dynamics of two spatially separated systems, focusing on the resulting loss of entanglement.

The main result of this paper is that a physical mechanism, based on a microscopic model of interactions between charges and phonons in solids and quantitatively confirmed by experiments on real quantum dot systems, may lead to the *complete decay* of the entanglement of a pair of two-level systems. This is in striking contrast with the partial asymptotic decay of local coherence under the same environmental dephasing. Moreover, using the physical coupling constants for spatially localized states we are able to describe the effect of spatial separation on the evolution of entanglement and to physically account for the crossover from a collective to an individual reservoir regime. These results are of importance not only for the general understanding of the properties of entanglement of open systems but also for practical tasks related to coherent control of charge states in multi-partite semiconductor systems (like carrier states in coupled quantum dots). This, in turn, may affect the feasibility of solid-state based quantum computing schemes [21, 22], including quantum error correction based on collective encoding of logical qubits (concatenation) [23].

The paper is organized as follows. In Sec. II we define the model of the excitonic two-qubit system and find its

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evolution. Next, in Sec. III, we discuss the evolution of entanglement between the two qubits. Sec. IV concludes the paper with final remarks.

## II. THE MODEL AND EVOLUTION OF THE TWO-QUBIT SYSTEM

In this Section we describe the evolution of the two excitonic qubits under the dephasing effect of phonons. The result will be used in the next Section for the calculation of entanglement between the two qubits.

The system is described by the Hamiltonian

$$\begin{aligned} H = & \epsilon_1(|1\rangle\langle 1| \otimes \mathbb{I}) + \epsilon_2(\mathbb{I} \otimes |1\rangle\langle 1|) + \Delta\epsilon(|1\rangle\langle 1| \otimes |1\rangle\langle 1|) \\ & + (|1\rangle\langle 1| \otimes \mathbb{I}) \sum_{\mathbf{k}} f_{\mathbf{k}}^{(1)}(b_{\mathbf{k}}^\dagger + b_{-\mathbf{k}}) \\ & + (\mathbb{I} \otimes |1\rangle\langle 1|) \sum_{\mathbf{k}} f_{\mathbf{k}}^{(2)}(b_{\mathbf{k}}^\dagger + b_{-\mathbf{k}}) + \sum_{\mathbf{k}} \omega_{\mathbf{k}} b_{\mathbf{k}}^\dagger b_{\mathbf{k}}, \end{aligned} \quad (1)$$

where the two states of each subsystem are denoted by  $|0\rangle$  and  $|1\rangle$ ,  $\mathbb{I}$  is the unit operator,  $\epsilon_{1,2}$  are the transition energies in the two subsystems,  $\Delta\epsilon$  is an energy shift due to the interaction between the subsystems,  $f_{\mathbf{k}}^{(1,2)}$  are system-reservoir coupling constants,  $b_{\mathbf{k}}, b_{\mathbf{k}}^\dagger$  are bosonic operators of the reservoir modes, and  $\omega_{\mathbf{k}}$  are the corresponding energies (we put  $\hbar = 1$ ). The explicit tensor notation refers to the two subsystems but is suppressed for the reservoir components.

Exciton wave functions will be modelled by anisotropic Gaussians with the extension  $l_{e/h}$  in the  $xy$  plane for the electron/hole and  $l_z$  along  $z$  for both particles. Then, the coupling constants for the deformation potential coupling between confined charges and longitudinal phonon modes have the form  $f_{\mathbf{k}}^{(1,2)} = f_{\mathbf{k}} e^{\pm i k_z d/2}$ , where  $d$  is the distance between the subsystems ( $d = 0$  corresponds to a biexciton, i.e., two excitons with different spins in a single QD) and  $f_{\mathbf{k}} = f_{\mathbf{k}}^{(e)} - f_{\mathbf{k}}^{(h)}$ , with

$$f_{\mathbf{k}}^{(e/h)} = \sigma_{e/h} \sqrt{\frac{k}{2\rho V c}} \exp\left[-\frac{l_z^2 k_z^2 + l_{e/h}^2 k_\perp^2}{4}\right].$$

Here  $V$  is the normalization volume of the bosonic reservoir,  $k_{\perp/z}$  are momentum components in the  $xy$  plane and along the  $z$  axis,  $\sigma_{e/h}$  are deformation potential constants for electrons/holes,  $c$  is the speed of longitudinal sound, and  $\rho$  is the crystal density. In our calculations we put  $\sigma_e = 8$  eV,  $\sigma_h = -1$  eV,  $c = 5.6$  nm/ps,  $\rho = 5600$  kg/m<sup>3</sup> (corresponding to GaAs), and  $l_e = 4.4$  nm,  $l_h = 3.6$  nm,  $l_z = 1$  nm.

Although the above coupling constants correspond to a specific system, their form reflects rather general physical conditions. First, for any translationally invariant reservoir,  $\mathbf{k}$  may be interpreted as momentum. Second, the system is assumed to be localized in space and to interact with the reservoir via a spatially local interaction (this is natural for solid state systems and corresponds to

the dipole approximation of electrodynamics). Then, if the system size is  $l$  the uncertainty of momentum is  $\sim 1/l$  and momentum conservation allows coupling to bosonic modes only within this range of  $\mathbf{k}$ , as manifested by the Gaussian cutoff in our formula. The position-dependent phase is also a general feature, while the analytical behavior at  $k \rightarrow 0$  is specific to the coupling.

The Hamiltonian (1) is diagonalized by the transformation  $\mathbb{W} = \sum_{i=1}^3 |i\rangle\langle i| W_i$ , where we use the standard product basis  $|0\rangle \equiv |0\rangle|0\rangle$ ,  $|1\rangle \equiv |0\rangle|1\rangle$ , etc., and  $W_i$  are Weyl shift operators

$$W_i = \exp\left[\sum_{\mathbf{k}} g_{\mathbf{k}}^{(i)*} b_{\mathbf{k}} - \text{H.c.}\right],$$

where  $g_{\mathbf{k}}^{(1,2)} = f_{\mathbf{k}}^{(1,2)}/\omega_{\mathbf{k}}$  and  $g_{\mathbf{k}}^{(3)} = g_{\mathbf{k}}^{(1)} - g_{\mathbf{k}}^{(2)}$ . Upon this unitary transformation one gets

$$\tilde{H} = \mathbb{W} H \mathbb{W}^\dagger = \tilde{H}_L + \tilde{H}_I + \tilde{H}_{\text{res}},$$

where

$$\tilde{H}_L = E_1(|1\rangle\langle 1| \otimes \mathbb{I}) + E_2(\mathbb{I} \otimes |1\rangle\langle 1|)$$

describes the independent (local) evolution of the subsystems,

$$\tilde{H}_I = \Delta E(|1\rangle\langle 1| \otimes |1\rangle\langle 1|)$$

describes their interaction, and

$$\tilde{H}_{\text{res}} = \sum_{\mathbf{k}} \omega_{\mathbf{k}} b_{\mathbf{k}}^\dagger b_{\mathbf{k}}$$

is the reservoir Hamiltonian. The energies here are

$$E_i = \epsilon_i - \sum_{\mathbf{k}} \omega_{\mathbf{k}} |g_{\mathbf{k}}^{(i)}|^2$$

and

$$\Delta E = \Delta\epsilon - 2 \text{Re} \sum_{\mathbf{k}} \omega_{\mathbf{k}} g_{\mathbf{k}}^{(1)} g_{\mathbf{k}}^{(2)*}.$$

The evolution operator generated by the Hamiltonian (1) may now be written as  $U_t = \mathbb{W}^\dagger \mathbb{W}_t \tilde{U}_t$ , where  $\tilde{U}_t = \exp(-i\tilde{H}t)$  and  $\mathbb{W}_t = \tilde{U}_t \mathbb{W} \tilde{U}_t^\dagger$ . Since  $\tilde{H}$  is diagonal, the evolution described by  $\tilde{U}_t$  is trivial. The final formulas are further simplified by performing the local unitary rotation  $U_L = \exp(i\tilde{H}_L t)$  (note that  $[U_L, \mathbb{W}_t] = 0$ ). Then, for the reduced density matrix of the two-qubit system one finds  $\rho(t) = U_L^\dagger \tilde{\rho}(t) U_L$ , with

$$\tilde{\rho}(t) = \text{Tr}_R \left[ \mathbb{W}^\dagger \mathbb{W}_t e^{-i\tilde{H}_I t} (\rho_0 \otimes \rho_T) e^{i\tilde{H}_I t} \mathbb{W}_t^\dagger \mathbb{W} \right], \quad (2)$$

where we assumed that the system is initially in a product state with the reservoir in the thermal equilibrium state  $\rho_T$ . Since local unitary transformations do not change the amount of entanglement, we may use the density matrix  $\tilde{\rho}(t)$  instead of  $\rho(t)$  in the calculations.

The elements of the density matrix  $\tilde{\rho}(t)$  are found using Eq. (2), the definition of the operator  $\mathbb{W}$ , and rules for multiplying and averaging Weyl operators [19, 24]. The result may be written in the operator sum representation

$$\tilde{\rho}(t) = \sum_{\mu} U K_{\mu} \tilde{\rho}(0) K_{\mu}^{\dagger} U^{\dagger} \quad (3)$$

with the unitary operator

$$U = |0\rangle\langle 0| + \exp \left[ i \sum_{\mathbf{k}} |g_{\mathbf{k}}|^2 \sin \omega_{\mathbf{k}} t \right] (|1\rangle\langle 1| + |2\rangle\langle 2|) \\ + \exp \left[ 4i \sum_{\mathbf{k}} |g_{\mathbf{k}}|^2 \cos^2 \frac{k_z d}{2} \sin \omega_{\mathbf{k}} t - i \Delta E t \right] |3\rangle\langle 3|$$

and the set of Kraus operators

$$K_0 = \text{diag}[a(t), b(t), b(t), a(t)], \\ K_1 = \text{diag}[(a^2(t) - 1)\sqrt{a^2(t) + 1}, 0, 0, 0], \\ K_2 = \sqrt{1 - a^2(t)} \text{diag}[-a^2(t), 0, 0, 1], \\ K_3 = \text{diag}[0, 0, (b^2(t) - 1)\sqrt{b^2(t) + 1}, 0], \\ K_4 = \sqrt{1 - b^2(t)} \text{diag}[0, 1, -b^2(t), 0],$$

where

$$a(t) = \exp \left[ \sum_{\mathbf{k}} |g_{\mathbf{k}}|^2 \cos^2 \frac{k_z d}{2} (\cos \omega_{\mathbf{k}} t - 1)(2n_{\mathbf{k}} + 1) \right] \quad (4a) \\ b(t) = \exp \left[ \sum_{\mathbf{k}} |g_{\mathbf{k}}|^2 \sin^2 \frac{k_z d}{2} (\cos \omega_{\mathbf{k}} t - 1)(2n_{\mathbf{k}} + 1) \right] \quad (4b)$$

and  $g_{\mathbf{k}} = f_{\mathbf{k}}/\omega_{\mathbf{k}}$ . The set of operators  $K_{\mu}$  corresponds to a kind of two-qubit phase damping channel. This channel is generated by the same physical pure dephasing process that would lead to the usual phase-damping channel for a single qubit [25]. The operators  $K_{1,2}$  may be interpreted as an effect of a “charge detector” which is sensitive only to the total number of excitons in the system. On the other hand,  $K_{3,4}$  reflect the action of a “discriminator” which detects in which of the two dots the exciton is present. Since the two subsystems are identical they may be distinguished by the reservoir only because of their different position. Therefore, the latter contribution to dephasing is inefficient for  $d = 0$ . For a system of two excitons confined in a single dot ( $d = 0$ ) the present result reduces to that found in Ref. [20].

The dephasing action of the reservoir develops in the course of the joint carrier-phonon evolution. Initially, all the Kraus operators except for  $K_0$  are null. For long times, the factors  $\cos \omega_{\mathbf{k}} t$  in Eqs. (4a,b) become quickly oscillating functions of  $\mathbf{k}$  and their contribution averages to 0. Consequently, the operators  $K_{\mu}$  reach a certain asymptotic form. As a result, the non-diagonal elements of the density matrix decrease from their initial value to a certain asymptotic value depending on material parameters, system geometry and temperature (see Fig. 1).

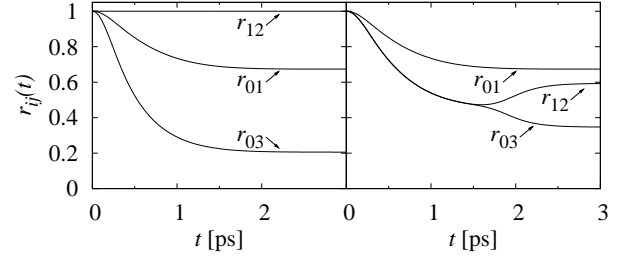


FIG. 1: The evolution of a two-qubit state under phonon-induced pure dephasing. The plot shows the relative reduction of the non-diagonal elements of the reduced density matrix,  $r_{ij} = |\rho_{ij}(t)/\rho_{ij}(0)|$  for  $T = 40$  K,  $d = 0$  (left), and  $d = 6$  nm (right). For identical qubits one has  $r_{01} = r_{02} = r_{13} = r_{23}$ .

For the actual carrier-phonon couplings in a semiconductor system this asymptotic value is *nonzero*, similarly as in the single-qubit dephasing [17] and the phase information is only partly erased. The non-monotonous evolution of coherence is due to the fact that the process is non-Markovian and some coherence may be regained in the course of system-reservoir interaction before the process is completed. It is also clear from Fig. 1 that the coherence between the states  $|1\rangle \equiv |0\rangle|1\rangle$  and  $|2\rangle \equiv |1\rangle|0\rangle$  may be broken only if the reservoir can distinguish between the two systems (i.e., for  $d \neq 0$ ).

In order to interpret the time dependence of the matrix elements for  $d \neq 0$ , shown in Fig. 1 (right), let us recall that the physical mechanism of dephasing is the emission of phonons from the excitonic qubit to the bulk of the crystal [26, 27]. These phonons form a spherical wave packet around the quantum dot, extending with the speed of sound and thus carrying the information from the qubit to the outside world [24]. For short times  $t \lesssim d/c$  this reservoir perturbation pertaining to each qubit stays localized around this qubit (the phonon wave packets do not overlap) so that detecting the presence of a charge is as efficient as deciding in which dot it is localized. As a result, the coherences between the ground and two-exciton state (matrix element  $\rho_{03}$ ) and between the two single-exciton states ( $\rho_{12}$ ) are affected to the same extent. For longer times, the wave packets originating from the two qubits intersect which, on one hand, leads to positive interference and to increased efficiency of overall charge detection. This is reflected in the right panel of Fig. 1 by a step-wise drop of  $\rho_{03}$  at  $t \sim d/c$ . On the other hand, however, this partial overlap makes it harder to identify the origin of the perturbation, hence the “discriminator” measurement becomes less efficient. Therefore, at  $t \gtrsim d/c$  the inter-qubit coherence  $\rho_{12}$  becomes less affected.

### III. DISENTANGLEMENT BY PURE DEPHASING

For a quantitative description of the decay of entanglement, a measure of entanglement that may be calculated

from the system state is needed. For pure states, the von Neumann entropy of one subsystem [28] may be used but for mixed states there is no unique entanglement measure [29, 30]. In order to quantitatively describe entanglement of systems in mixed states one defines various quantities, which may give essentially different entanglement measures. Nonetheless, any such quantity must satisfy certain natural requirements in order to be useful as an entanglement measure: it must vanish for separable states and be non-zero for entangled ones; it must reduce to the von Neumann entropy for a pure state; it must be non-increasing under local operations (it is impossible to increase entanglement by manipulating only one subsystem). One choice is to use the *entanglement of formation* (EOF), defined as the ensemble average of the von Neumann entropy minimized over all ensemble preparations of the state [29, 30], and thus being a natural generalization of the von Neumann entropy to mixed states. Such a measure may be interpreted as the asymptotic number of pure singlets necessary to prepare the state by local operations and classical communication. A practical characterization for mixed state entanglement is available for small systems [31, 32] but an explicit formula for calculating an entanglement measure is known only for the EOF of a pair of two-level systems [33, 34].

We will perform the calculations for two initial pure states

$$|\psi_0^{(1)}\rangle = \frac{|00\rangle + |01\rangle + |10\rangle - |11\rangle}{2}, \quad (5a)$$

$$|\psi_0^{(2)}\rangle = \frac{|01\rangle - |10\rangle}{\sqrt{2}}. \quad (5b)$$

It is assumed that these states are prepared within a time shorter than reservoir response times. Both initial states (5a) and (5b) are maximally entangled, i.e. their entanglement of formation is equal to 1.

Using Eq. (3), we calculate the EOF of the system state at a time  $t$  from the Wootters formula [33, 34]

$$E[\rho(t)] = E[\tilde{\rho}(t)] = -x_+ \log_2 x_+ - x_- \log_2 x_-,$$

where

$$x_{\pm} = \frac{1 \pm \sqrt{1 - C^2[\tilde{\rho}(t)]}}{2}$$

and  $C[\tilde{\rho}(t)]$  is the concurrence, given by

$$C[\tilde{\rho}(t)] = \max(0, \lambda_0 - \lambda_1 - \lambda_2 - \lambda_3), \quad (6)$$

where  $\lambda_i$  are the eigenvalues of the matrix  $\tilde{\rho}(t)(\sigma_y \otimes \sigma_y)\tilde{\rho}(t)(\sigma_y \otimes \sigma_y)$  in decreasing order.

The evolution of the EOF of the qubit pair is shown in Fig. 2. In the absence of energy shift  $\Delta E$  (solid lines), entanglement decays on a time scale of a few picoseconds. At low temperatures or for overlapping systems, this process resembles the decay of coherences shown in Fig. 1. However, for a sufficiently large separation between the systems and at sufficiently high temperatures

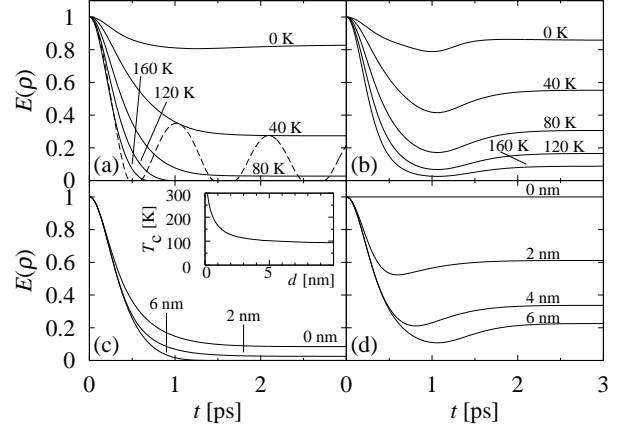


FIG. 2: Evolution of entanglement of the two-qubit system at various temperatures with  $d = 6$  nm (a,b) and for various distances  $d$  at  $T = 100$  K (c,d). The left panels (a,c) show the result for the initial state (5a) and the right ones (b,d) for the singlet state (5b). The inset in (c) shows the lowest temperature at which the decay becomes complete for a given distance. Solid lines correspond to  $\Delta E = 0$  and the dashed line to  $\Delta E/\hbar = 6$  ps $^{-1}$  (at  $T = 40$  K).

the initially maximal entanglement present in the state (5a) decays completely after a finite time even though the environment-induced dephasing is always only partial (see Fig. 2a,c). The temperature  $T_c$  at which the system becomes separable is related to the distance as shown in the inset in Fig. 2c. On the other hand, for the other initial state [Eq. (5b)], the destruction of entanglement is always only partial (Fig. 2b,d).

An important case is that of  $\Delta E \neq 0$ . Such an energy shift (known as the biexcitonic shift in a semiconductor system) leads to an entanglement-generating evolution. This mechanism is used for performing nontrivial two-qubit gates (controlled-shift) in many proposals for semiconductor-based quantum information processing [21, 35]. As can be seen in Fig. 2a (dashed line), in the presence of phonon-induced pure dephasing the cyclic evolution of entanglement is damped and the maximum achievable level of entanglement is reduced. Moreover, extended periods of time appear when the entanglement remains zero.

The appearance of complete disentanglement for some initial states under sufficiently strong partial pure dephasing may be understood with the help of Eq. (6). If the completely dephased state (with a diagonal density matrix) has  $\lambda_0 - \lambda_1 - \lambda_2 - \lambda_3 < 0$  then, by continuity, it will be surrounded by states with vanishing concurrence, thus separable. In this case entanglement vanishes for sufficiently strongly dephased states, before the complete dephasing is reached. For a diagonal density matrix one finds  $\lambda_0 - \lambda_1 - \lambda_2 - \lambda_3 = -2 \min(\rho_{00}\rho_{33}, \rho_{11}\rho_{22})$ , so that the above condition may only be satisfied if all four diagonal elements are nonzero. For instance, the totally mixed state (with a density matrix proportional to unity) is surrounded by a ball of separable states [36]. Out of the

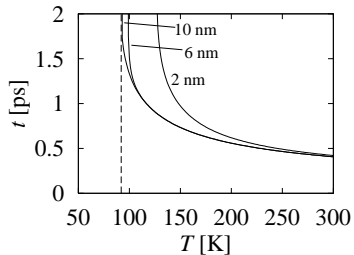


FIG. 3: The time at which the entanglement of the initial state (5a) decays completely as a function of temperature.

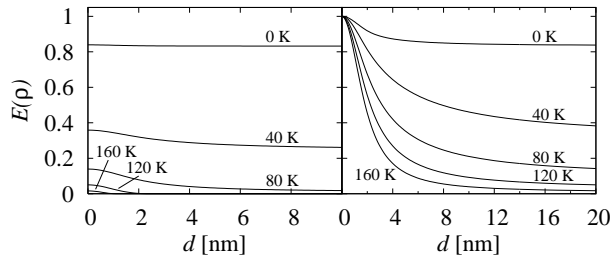


FIG. 4: The asymptotic value of the EOF as a function of the distance between the subsystems. Left and right panels correspond to the initial states (5a) and (5b), respectively.

initial states considered here, the first one [Eq. (5a)] satisfies the above condition (it decays towards the totally mixed state) but the second one [Eq. (5b)] does not. In general, the structure of entangled and separable states around the final state may be quite complicated.

The time at which the entanglement of the state (5a) vanishes completely depends on temperature and on the distance between the systems. As can be seen in Fig. 3, this time becomes finite only at a certain temperature (dashed line in the figure). Slightly above this critical temperature, complete disentanglement takes place only for strongly separated systems. For higher temperatures the disentanglement time for non-overlapping systems depends very weakly on the distance. It should be stressed that the appearance of complete disentanglement at increased temperatures is only related to stronger dephasing and, in principle, the state might become separable already at  $T = 0$  if the coupling were sufficiently strong.

On the other hand, the dependence on the separation between the two subsystems reflects a more fundamental crossover, from the regime of a common reservoir to that of independent reservoirs [9]. In our model, the non-diagonal element  $\rho_{12}$  is unaffected for completely overlapping systems ( $d = 0$ ) (see Fig. 1). This single non-vanishing element is enough to prevent complete dis-

entanglement at any temperature and for any coupling strength. Only when the distance between the systems grows, dephasing is able to decrease this element to the extent sufficient for the total destruction of entanglement. This distance effect is shown in Fig. 4, where we plot the asymptotic (long-time) value of the EOF.

The effect of spatial separation between the systems is very clearly visible for the second initial state [Eq. (5b)], which involves *only* this robust non-diagonal element (see Fig. 4, right). Here, the entanglement is absolutely stable if the systems overlap (see also Fig. 2d) but becomes much more fragile as soon as the separation between the systems is comparable to their size. This demonstrates that the distance between the subsystems is the physical parameter that governs the crossover between the two regimes of entanglement decay.

#### IV. CONCLUSIONS

We have studied the effect of pure dephasing on the entanglement of two-level subsystems. We have shown that partial dephasing may be sufficient to completely destroy entanglement for a class of initial states that may be easily characterized. Apart from its dependence on the initial state and temperature, the disentanglement effect shows essential dependence on the distance between the subsystems, manifesting a crossover between two regimes of reservoir-induced dephasing. Complete disentanglement appears only for spatially separated systems.

Complete disentanglement due to partial pure dephasing typical for localized carrier states in semiconductor systems is not only of general interest but also of relevance to solid state implementations of quantum information processing. Moreover, understanding the role of the distance between subsystems in maintaining quantum correlations between them seems to be essential for realistic design of quantum error correction schemes based on collective encoding of logical qubits (concatenation).

One should note that, due to a purely non-Markovian character of the dephasing effect, the system evolution depends on the way in which the “initial” state has been prepared from the ground system state. Therefore, a reduction of the destructive effect may be expected if the preparation is done either slowly (adiabatically) or by shaped pulses [37, 38].

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- [1] A. Einstein, B. Podolsky, and N. Rosen, Phys. Rev. **47**, 777 (1935).
  - [2] E. Schrödinger, Proc. Cambridge Philos. Soc. **31**, 555 (1935).
  - [3] J. S. Bell, Physics **1**, 195 (1964).

- [4] M. A. Nielsen and I. L. Chuang, *Quantum Computation and Quantum Information* (Cambridge University Press, Cambridge, 2000).
- [5] C. H. Bennett, G. Brassard, C. Crépeau, R. Jozsa, A. Peres, and W. K. Wootters, Phys. Rev. Lett. **70**, 1895 (1993).

- (1993).
- [6] C. H. Bennett and S. J. Wiesner, Phys. Rev. Lett. **69**, 2881 (1992).
  - [7] A. K. Ekert, Phys. Rev. Lett. **67**, 661 (1991).
  - [8] T. Yu and J. H. Eberly, Phys. Rev. B **66**, 193306 (2002).
  - [9] T. Yu and J. H. Eberly, Phys. Rev. B **68**, 165322 (2003).
  - [10] T. Yu and J. H. Eberly, Phys. Rev. Lett. **93**, 140404 (2004).
  - [11] P. J. Dodd and J. J. Halliwell, Phys. Rev. A **69**, 052105 (2004).
  - [12] L. Diósi, in *Irreversible Quantum Dynamics (Lecture Notes in Physics vol. 622)*, edited by F. Benatti and R. Floreanini (Springer, Berlin, 2003), pp. 157–163, quant-ph/0301096.
  - [13] M. Horodecki, P. Horodecki, and R. Horodecki, Phys. Rev. Lett. **78**, 574 (1997).
  - [14] A. Vagov, V. M. Axt, T. Kuhn, W. Langbein, P. Borri, and U. Woggon, Phys. Rev. B **70**, 201305(R) (2004).
  - [15] H.-P. Breuer and F. Petruccione, *The Theory of Open Quantum Systems* (Oxford University Press, Oxford, 2002).
  - [16] P. Borri, W. Langbein, S. Schneider, U. Woggon, R. L. Sellin, D. Ouyang, and D. Bimberg, Phys. Rev. Lett. **87**, 157401 (2001).
  - [17] B. Krummheuer, V. M. Axt, and T. Kuhn, Phys. Rev. B **65**, 195313 (2002).
  - [18] R. Alicki, Open Sys. and Information Dyn. **11**, 53 (2004).
  - [19] G. D. Mahan, *Many-Particle Physics* (Kluwer, New York, 2000).
  - [20] V. M. Axt, T. Kuhn, A. Vagov, and F. M. Peeters, Phys. Rev. B **72**, 125309 (2005).
  - [21] E. Biolatti, R. C. Iotti, P. Zanardi, and F. Rossi, Phys. Rev. Lett. **85**, 5647 (2000).
  - [22] T. Calarco, A. Datta, P. Fedichev, E. Pazy, and P. Zoller, Phys. Rev. A **68**, 12310 (2003).
  - [23] E. Knill, Nature **434**, 39 (2005).
  - [24] K. Roszak and P. Machnikowski, cond-mat/0504135, to appear in Phys. Lett. A.
  - [25] J. Preskill, lecture notes available at <http://www.theory.caltech.edu/~preskill/ph229> (unpublished).
  - [26] L. Jacak, P. Machnikowski, J. Krasnyj, and P. Zoller, Eur. Phys. J. D **22**, 319 (2003).
  - [27] A. Vagov, V. M. Axt, and T. Kuhn, Phys. Rev. B **66**, 165312 (2002).
  - [28] C. H. Bennett, H. J. Bernstein, S. Popescu, and B. Schumacher, Phys. Rev. A **53**, 2046 (1996).
  - [29] C. H. Bennett, G. Brassard, S. Popescu, B. Schumacher, J. A. Smolin, and W. K. Wootters, Phys. Rev. Lett. **76**, 722 (1996).
  - [30] C. H. Bennett, D. P. DiVincenzo, J. A. Smolin, and W. K. Wootters, Phys. Rev. A **54**, 3824 (1996).
  - [31] A. Peres, Phys. Rev. Lett. **77**, 1413 (1996).
  - [32] M. Horodecki, P. Horodecki, and R. Horodecki, Phys. Lett. A **223**, 1 (1996).
  - [33] S. Hill and W. K. Wootters, Phys. Rev. Lett. **78**, 5022 (1997).
  - [34] W. K. Wootters, Phys. Rev. Lett. **80**, 2245 (1998).
  - [35] E. Pazy, E. Biolatti, T. Calarco, I. D’Amico, P. Zanardi, F. Rossi, and P. Zoller, Europhys. Lett. **62**, 175 (2003).
  - [36] K. Życzkowski, P. Horodecki, A. Sanpera, and M. Lewenstein, Phys. Rev. A **58**, 883 (1998).
  - [37] R. Alicki, M. Horodecki, P. Horodecki, R. Horodecki, L. Jacak, and P. Machnikowski, Phys. Rev. A **70**, 010501(R) (2004).
  - [38] V. M. Axt, P. Machnikowski, and T. Kuhn, Phys. Rev. B **71**, 155305 (2005).